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Boundary Conditions for the Vorticity–Velocity Formulation of Navier–Stokes Equations

Alexander L. Ni*
Asea Brown Boveri, Ltd., CH-5405 Baden, Switzerland

Introduction

THE no-slip boundary conditions for the vorticity–velocity formulation or, more precisely, the $u - \omega$ integro-differential formulation of the Navier–Stokes (NS) equations are considered in this paper. The status of the problem has been given in the recent reviews by Gresho[1] and Gatski.[2] The present Note is concerned with the no-slip boundary conditions within the framework of the boundary-element method that has been in development for the past two decades by Wu,[3,4,8] Wu and Gulcat,[5] Wu and Thomson,[6] and Wang and Wu.[7] This method has appeared to be rather fruitful. Important problems for both practice and basic research have been solved with this method. Wu et al.[3–8] presented the equation that is sometimes used to represent a solution on NS equations for given time instant in integral form. "Sometimes" is written here because the solver given by those authors does not generally speaking meet the no-slip boundary conditions. Gresho[1] wrote in his review that under some circumstances the equation "... may not be strictly valid, and some mathematical trickery seems to have crept in. Woe is Wu? No, because in practice it is quite properly (and cleverly) utilized in other ways, and the suggested illegitimacy is in fact vindicated." However, the problem for the general case remains unsolved. How to meet the no-slip conditions? This Note approaches these questions by reduction of the problem finally to the system of Fredholm equations of the second kind. The solver of this system gives the distribution of vortices and sources (sinks) on the surface of the body. The integral representation of the solution with the obtained vortices and sources yields the exact instantaneous velocity field that meets the no-slip conditions.

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*Guest Scientist, CRBT; currently Principal Researcher, Institute of Chemical Physics, 142432 Chernogolovka, Moscow, Russia.

Flow Equations

The terminology and notations from Ref. 1 are used throughout this paper. The governing equations for an incompressible viscous medium are

$$\nabla \boldsymbol{u} = 0, \qquad \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\nabla)\boldsymbol{u} = \eta \Delta \boldsymbol{u} - \nabla P \tag{1}$$

The most general initial-boundary-value problem (IBVP) is as follows:[1] Find the velocity $\boldsymbol{u}(x, t)$ and the kinematic pressure $P(x, t)$, i.e., the pressure divided by the density in a bounded domain Ω with a boundary value of $\partial\Omega = \Gamma = \Gamma_D + \Gamma_N$, subject to the boundary conditions for $t > 0$ of

$$\boldsymbol{u} = \boldsymbol{w} \qquad \text{on} \qquad \Gamma_D \tag{2}$$

$$-P + \eta\, \partial u_n/\partial n = 0, \qquad \eta\, \partial u_\tau/\partial n = 0 \qquad \text{on} \qquad \Gamma_N \tag{3}$$

where n indicates the normal component and outward normal direction, t indicates the tangential direction, and wherein if (and only if) $\Gamma = \Gamma_D(\Gamma_N)$, for $\Gamma_N = \emptyset$, $\boldsymbol{w}$ must satisfy $\oint_\Gamma \boldsymbol{n}\boldsymbol{w} = 0$ for $t > 0$, and is subject to the initial condition $\boldsymbol{u}(x, 0) = \boldsymbol{u}_0(x)$ in Ω, where $\nabla \boldsymbol{u}_0 = 0$ in Ω, and $\boldsymbol{n}\boldsymbol{u}_0 = \boldsymbol{n}\boldsymbol{w}_0 = \boldsymbol{n}\boldsymbol{w}(x, t)$, $(x \in \Gamma$ and $t = 0)$ on Γ_D.

Apply the curl operator $\nabla \times (\cdots)$ to Eqs. (1) and use the fact that both $\boldsymbol{u}$ and ω are division-free to obtain

$$\frac{\partial \omega}{\partial t} + (\boldsymbol{u}\nabla)\omega = (\omega\nabla)\boldsymbol{u} + \eta\Delta\omega \tag{4a}$$

and the velocity vector

$$\omega = \nabla \times \boldsymbol{u} = \operatorname{rot} \boldsymbol{u} \tag{4b}$$

Vector-Potential-Vorticity ($A - \omega$) Formulation

If one now focuses on the no-slip boundary condition Eq. (2) the vector-potential A and the scalar potential φ by means of

$$\boldsymbol{u} = \operatorname{rot} \boldsymbol{A} + \nabla\varphi \tag{5}$$

to arrive at the second in the pair known as the vector-potential-vorticity ($\boldsymbol{A} - \omega$) formulation

$$\Delta \boldsymbol{A} = -\omega, \qquad \Delta\varphi = 0 \tag{6}$$

It is important that one additional requirement (a gauge condition) be imposed on A for the derivation of Eqs. (6):

$$\operatorname{div} \boldsymbol{A} = 0 \tag{7}$$

The no-slip boundary conditions on the surface of the body in the absolute frame are

$$(\nabla\varphi)_n + (\operatorname{rot}\boldsymbol{A})_n - (\boldsymbol{w})_n = 0, \qquad (\nabla\varphi)_t + (\operatorname{rot}\boldsymbol{A})_t - (\boldsymbol{w})_t = 0 \tag{8}$$

The latter equation is really a vector equation on the surface.

The general representations of the solutions for the Poisson equation for the vector potential and the Laplace equation for the scalar potential (6), are[9]

$$\begin{aligned} \boldsymbol{A}(\boldsymbol{x}, t) &= \boldsymbol{\Phi}(\boldsymbol{x}) + \int_\Gamma \mu(\boldsymbol{x}')F(s)\,\mathrm{d}\Gamma(\boldsymbol{x}') \\ \varphi(\boldsymbol{x}, t) &= \int_\Gamma \nu(\boldsymbol{x}')F(s)\,\mathrm{d}\Gamma(\boldsymbol{x}') \end{aligned} \tag{9}$$

where the vector $\boldsymbol{\Phi}(\boldsymbol{x}) = -\beta_d \int_\Omega \omega(\boldsymbol{x}')F(s)\,\mathrm{d}\Omega(\boldsymbol{x}')$.

The functions $\mu(\boldsymbol{x}')$ and $\nu(\boldsymbol{x}')$, entered in the surface integrals, denote the surface vorticity and the surface sources (sinks), respectively. They should be determined from the boundary conditions. Also here, $F(s)$ is the fundamental solution of the Laplace equation, where $F(s) = -1/s$, $\beta_d = 1/4\pi$ for three dimensions, $F(s) = -\ln s$, $\beta_d = 1/2\pi$ for two dimensions, $\boldsymbol{s} = \boldsymbol{x} - \boldsymbol{x}'$, and $s = |\boldsymbol{s}|$.

Substituting Eqs. (9) into Eq. (5) yields the following formula for the velocity:

$$\boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{u}_\omega(\boldsymbol{x}, t) + \nabla \int_\Gamma \nu(\boldsymbol{x}') F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') + \mathrm{rot} \int_\Gamma \boldsymbol{\mu}(\boldsymbol{x}') F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') \tag{10}$$

where the vector $\boldsymbol{u}_\omega(\boldsymbol{x}, t) = \mathrm{rot}\, \boldsymbol{\Phi}(\boldsymbol{x})$ is by definition the velocity induced by the distributed vorticity at a given point of space.

Under the assumption that the differential operators and integrals commute (let us emphasize at this point that this assumption is not valid for the points belonging to the surface!), the latter formula can be reduced to the form that seems to be similar to that given by Wu et al.[3–8]:

$$\boldsymbol{u}(\boldsymbol{x}, t) = -\beta_d \left[\int_\Omega \frac{\boldsymbol{\omega}(\boldsymbol{x}') \cdot \boldsymbol{s}}{s^d}\, \mathrm{d}\Omega(\boldsymbol{x}') + \int_\Gamma \frac{\boldsymbol{n} \cdot \boldsymbol{w}(\boldsymbol{x}')\boldsymbol{s}}{s^d}\, \mathrm{d}\Gamma(\boldsymbol{x}') + \int_\Gamma \frac{\boldsymbol{n} \cdot \boldsymbol{w}(\boldsymbol{x}') \cdot \boldsymbol{s}}{s^d}\, \mathrm{d}\Gamma(\boldsymbol{x}') \right]$$

Distinct from Eq. (10), the formula developed by Wu and others expresses the surface vorticity and sources (sinks) by means of the unknown velocity vector $\boldsymbol{w}(\boldsymbol{x}')$. The problem of the determination of $\boldsymbol{w}(\boldsymbol{x}')$ was discussed thoroughly in the papers by Wu et al.[3–8] According to Ref. 8, the correct boundary condition for the problem is either the normal velocity component $\boldsymbol{n} \cdot \boldsymbol{w}(\boldsymbol{x}')$ or the tangential velocity $\boldsymbol{n} \cdot \boldsymbol{w}(\boldsymbol{x}')$ or it can be a linear combination of the two over the entire boundary. These boundary conditions are at best the approximation to the exact no-slip conditions. The accuracy of this approximation cannot be estimated a priori. The exact no-slip boundary conditions should be considered on the basis of the representation in Eq. (10). (Consider the two- and three-dimensional cases separately.)

Two-Dimensional Flow

For the two-dimensional flow in the x, y plane, the vector potential can be taken in a form with only one nonzero z component $A_z = \psi$, where ψ is the stream function. The velocity and the vorticity components (the latter has one nonzero component ($\omega_z = \omega$) expressed by means of ψ and φ are

$$u = \partial\psi/\partial y + \partial\varphi/\partial x, \qquad v = -\partial\psi/\partial x + \partial\varphi/\partial y$$

$$\omega = \partial v/\partial x - \partial u/\partial y$$

The boundary conditions derived in Eqs. (8) in terms of φ and ψ have the following form:

$$\varphi_n + \psi_t - w_n = 0, \qquad \varphi_t - \psi_n - w_t = 0 \tag{11}$$

The general solution of the problem is given by formulas (9).

As was mentioned above, for the points in Ω, the differential operators in Eq. (10) can be applied directly to the expressions inside the integrals. The modification of this formula for the points belonging the boundary Γ needs separate consideration. The theory of harmonic functions[9] for this case leads to the equation

$$\frac{\partial}{\partial n} \oint_{\Gamma'} \nu(x', y') \ln[(x - x')^2 + (y - x')^2]\, \mathrm{d}\Gamma = 2\pi\nu(x, y) + \oint_{\Gamma'} \nu(x', y') \frac{\partial}{\partial n} \ln[(x - x')^2 + (y - x')^2]\, \mathrm{d}\Gamma$$

where $\partial/\partial n$ is the operator of the differentiation in the direction of the outer normal $\boldsymbol{n}$ to the surface at the point (x, y). Taking this differentiation rule into account one comes to the following representation of the boundary conditions of Eqs. (11):

$$\pi\nu(x, y) + \oint_{\Gamma'} \nu(x', y') \frac{sn}{s^2}\, \mathrm{d}\Gamma' + \oint_{\Gamma'} \mu(x', y') \frac{st}{s^2}\, \mathrm{d}\Gamma + u_{\omega n} - w_n = 0$$

$$-\pi\mu(x, y) - \oint_{\Gamma'} \mu(x', y') \frac{sn}{s^2}\, \mathrm{d}\Gamma' + \oint_{\Gamma'} \nu(x', y') \frac{st}{s^2}\, \mathrm{d}\Gamma + u_{\omega t} - w_t = 0$$

These equations represent the system of Fredholm integral equations of the second kind, incorporating the unknown functions μ and ν. After these functions are found, the velocity field given by formula (9), meets the exact no-slip boundary conditions. Together with these equations, the transport equation (4) represents a closed system of governing equations for the two-dimensional NS flows.

Three-Dimensional Flow

Equation (7) for the two-dimensional case has been satisfied automatically by the choice of $\boldsymbol{A}$ as having only one nonzero component. For the three-dimensional case, Eq. (7) supplements one additional equation for the surface vorticity distribution $\boldsymbol{\mu}(\boldsymbol{x}')$. The necessity for the existence of one additional equation follows from the simple comparison of the number of boundary conditions with the number of unknown functions [there are three equations for the three velocity components and four unknown functions ($\nu(\boldsymbol{x}')$ and $\boldsymbol{\mu}(\boldsymbol{x}')$].

To derive the additional equation for the surface vorticity, let us introduce the new dependent variable $\varepsilon = \mathrm{div}\boldsymbol{A}$. If the operator div is applied to Eqs. (6), commuting the differential operators and taking into account that $\mathrm{div}\, \boldsymbol{\omega} = 0$ leads to the Laplace equation for ε in Ω:

$$\Delta\varepsilon = 0$$

Now let us derive the boundary conditions for ε. Calculations of the corresponding derivatives from Eqs. (9) yield

$$\varepsilon(\boldsymbol{x}, t) = -\beta_d \int_\Gamma \boldsymbol{\omega}(\boldsymbol{x}')\boldsymbol{n}' F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') + \mathrm{div} \times \left[\int_\Gamma \boldsymbol{\mu}(\boldsymbol{x}') F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') \right]$$

To find ε at the point $\boldsymbol{x}_0$ on the surface, we introduce at this point the local Cartesian coordinate systems with axes $\boldsymbol{n}_0, \boldsymbol{t}_{01}, \boldsymbol{t}_{02}$, where $\boldsymbol{n}_0$ is normal to Γ at $\boldsymbol{x}_0$ and $\boldsymbol{t}_{01}$ and $\boldsymbol{t}_{02}$ are vectors tangential to Γ. Expressed in the local coordinate system the divergence is

$$\varepsilon(\boldsymbol{x}_0, t) = -\beta_d \int_\Gamma \boldsymbol{\omega}(\boldsymbol{x}')\boldsymbol{n}' F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') + \frac{\partial}{\partial n_0} \int_\Gamma \mu_{n0}(\boldsymbol{x}') F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') + \mathrm{div}_t \int_\Gamma \boldsymbol{\mu}_t(\boldsymbol{x}') F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}')$$

The differentiation rule for the surface integrals[9] yields the following expression for the derivative along the outward normal direction n_0:

$$\frac{\partial}{\partial n_0} \int_\Gamma \mu_{n0}(\boldsymbol{x}') F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') = 2\pi\mu_{n0}(\boldsymbol{x}_0) + \int_\Gamma \frac{\mu_{n0}(\boldsymbol{x}')\boldsymbol{n}_0\boldsymbol{s}}{s^d}\, \mathrm{d}\Gamma(\boldsymbol{x}') \qquad \mu_{n0}(\boldsymbol{x}) = \boldsymbol{\mu}(\boldsymbol{x})\boldsymbol{n}_0$$

If ε equals zero on Γ, than it is zero everywhere in Ω and vice versa. It follows that $\boldsymbol{\mu}(\boldsymbol{x}')$ should satisfy the Fredholm integral equation of the second kind:

$$2\pi\mu_{n0}(\boldsymbol{x}) + \int_\Gamma \frac{\mu_{n0}(\boldsymbol{x}')\boldsymbol{n}_x\boldsymbol{s}}{s^d}\, \mathrm{d}\Gamma(\boldsymbol{x}') - \int_\Gamma \boldsymbol{\omega}(\boldsymbol{x}')\boldsymbol{n}' F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') + \mathrm{div}_t \left[\int_\Gamma \boldsymbol{\mu}_t(\boldsymbol{x}') F(s)\, \mathrm{d}\Gamma(\boldsymbol{x}') \right] = 0$$

relative to $\mu_{n0}(\boldsymbol{x})$.

Consideration of the boundary conditions represented by Eqs. (8) should be performed in a way similar to that shown above in the local coordinate system. The condition for the normal component yields

$$2\pi\nu(x) + \int_\Gamma \frac{\nu(x)n_x s}{s^d}\,\mathrm{d}\Gamma(x')$$

$$+ \mathrm{rot}_n\left[\Phi(x) + \int_\Gamma \mu(x')F(s)\,\mathrm{d}\Gamma(x')\right]w_n = 0$$

The boundary conditions for the tangential components are

$$2\pi\mu_{t_1}(x) + \int_\Gamma \frac{\mu_{t_1}(x')n_x s}{s^d}\,\mathrm{d}\Gamma(x') + \int_\Gamma \nu(x')\frac{\partial F(s)}{\partial t_2}\,\mathrm{d}\Gamma(x')$$

$$- \int_\Gamma \mu_n(x')\frac{\partial F(s)}{\partial t_1}\,\mathrm{d}\Gamma(x') + \mathrm{rot}_{t_2}\Phi(x) - w_{t_2} = 0$$

$$2\pi\mu_{t_2}(x) + \int_\Gamma \frac{\mu_{t_2}(x')n_x s}{s^d}\,\mathrm{d}\Gamma(x') - \int_\Gamma \nu(x')\frac{\partial F(s)}{\partial t_1}\,\mathrm{d}\Gamma(x')$$

$$- \int_\Gamma \mu_n(x')\frac{\partial F(s)}{\partial t_2}\,\mathrm{d}\Gamma(x') - \mathrm{rot}_{t_1}\Phi(x) + w_{t_1} = 0$$

In a manner similar to that of the two-dimensional case, the boundary-condition problem is reduced to the system of Fredholm equations of the second kind. This system, together with the vorticity transport equations, represents the closed system of governing equations for the three-dimensional NS flows.

Concluding Remarks

The intent of this Note has been to highlight some aspects concerning the no-slip boundary conditions for the vorticity–velocity formulation or, more precisely, the $u - \omega$ integro-differential formulation, of the NS equations. This Note has shown that the problem of the boundary condition can be reduced to the system of Fredholm equations of the second kind. The solution of this system gives the distribution of vortices and sinks on the surface of the body; the integral representation of the solution with vorticities and sources yields the exact solution that meets the no-slip conditions.

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Oblique-Shock/Expansion-Fan Interaction—Analytical Solution

H. Li* and G. Ben-Dor†
Ben-Gurion University of the Negev, Beer Sheva, Israel

Introduction

CONSIDER Fig. 1a, in which a supersonic flow having a Mach number M_0 flows inside a channel that suddenly turns by an angle δ_1. For the flow to negotiate this sudden turn, a centered expansion wave and an oblique shock wave are generated at the expansive and compressive corners, respectively, as shown in Fig. 1a. The oblique shock wave intersects the leading characteristic of the expansion fan at point A, initiating the interaction between these two waves. As a result, the slope of the oblique shock wave changes continuously until it emerges from the expansion fan at point B, which marks the intersection of the oblique shock wave with the tail of the expansion fan. Beyond point B the oblique shock wave is again straight. As shown in Fig. 1a it eventually reflects from the upper wall of the channel. Although the reflection shown in Fig. 1a is regular, a Mach reflection is also possible. The type of the reflection that actually occurs depends on both the aerodynamic and the geometric conditions. Upon crossing the curved portion of the oblique shock wave, the expansion waves are also bent, until they emerge from the slipstream and become straight. In Fig. 1a, the transmitted expansion waves are seen to reflect from the bottom wall of the channel.

Since the subject of this paper is the interaction between an oblique shock and a centered expansion fan, the subsequent interactions of the reflected shock and reflected expansion waves will not be considered here.

An interaction similar to the one shown in Fig. 1a takes place in a Mach reflection in steady flows, as shown in Fig. 1b. Here, the reflected shock wave R interacts with the centered expansion fan that is formed at the trailing edge of the reflecting wedge. The transmitted expansion waves interact then with the slipstream S and turns it away from the bottom surface. As a result a throat is formed in the flow tube formed by the bottom surface and the slipstream. According to Azevedo and Liu[1] the cross-sectional area of this throat determines the actual height of the Mach stem.

The interaction between an oblique shock and a centered expansion fan is observed in many other aerodynamic situations, e.g., a supersonic jet emanating from an overexpanded nozzle or the wave interaction associated with a ram accelerator, a device for accelerating projectiles to ultrahigh velocities. Surprisingly, however, it appears never to have been solved analytically. Consequently, the aim of the present study is such an analytical solution.

Present Study

The relevant flow states and flow parameters are defined in Fig. 1a together with the selected origin of the (x, y) coordinate system. State 0 is ahead of both the oblique shock and the centered expansion fan, state 2 is aft of the centered expansion fan, state 3 is aft of the transmitted oblique shock wave, and state 5 is obtained behind the oblique shock wave regularly reflected at point R. State 1 is reached just behind the oblique shock wave, and state 4 is aft of the transmitted expansion waves. Note that states 3 and 4, having different flow histories, are separated by a layer of varying entropy. The streamlines in these two flow states are parallel and the pressures are equal.

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*Ph.D. Student, Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering.

†Professor and Dean of Faculty of Engineering Sciences, Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering.